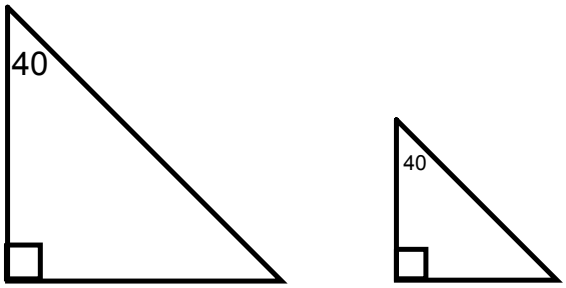
3/7/17 "Too many of us are not living our dreams because we are living our fears."-Les Brown

HW: "Related Rates" packet page 15 #2
Test 2 on Tuesday 3/14

AIM: How can we use relationships between variables to do related rates?

Warm Up:

1. What do we know about these triangles?



Page 8

1) A water tank in the shape of a right circular cone has a *height of 10 feet*. The top rim of the tank is a circle with *a radius of 4 feet*. If water is being pumped into the tank at the *rate of 2 cubic feet per minute*, what is the *rate of change of the water depth*, in feet per minute, when the *depth is 5 feet*?

① Know:

$h(\text{of tank}) = 10\text{ ft}$

$r(\text{of tank}) = 4\text{ ft}$

$\frac{dV}{dt} = 2\ \text{ft}^3/\text{min}$

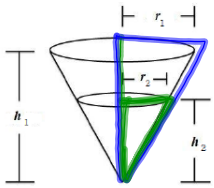
$h(\text{water}) = 5\text{ ft}$

② Need: $\frac{dh}{dt}$

③ $V = \frac{1}{3}\pi r^2 h$ ← Don't know / Can't find

$\frac{dV}{dt} = \frac{2}{3}\pi r\frac{dr}{dt}h + \frac{1}{3}\pi r^2\frac{dh}{dt}$

Cone



The relationship between the radius and height of the cone and the radius and height to the water:

$$\frac{r_1}{h_1} = \frac{r_2}{h_2}$$

The volume of a cone:

$$V = \frac{1}{3}\pi r^2 h$$

(where r is its radius and h is its height.)

$\frac{\text{radius of tank}}{\text{height of tank}} = \frac{\text{radius of top water}}{\text{height top water}}$

$\frac{4}{10} = \frac{r}{h}$

$\frac{4}{10}h = r$ ← Replace in the Volume Formula

New Equation:

$V = \frac{1}{3}\pi\left(\frac{4}{10}h\right)^2 \cdot h$

$V = \frac{1}{3}\pi\frac{16}{100}h^2 \cdot h$

$V = \frac{1}{3}\pi\frac{16}{100}h^3$

$V = \frac{16}{300}\pi h^3$

$\frac{dV}{dt} = \frac{48}{300}\pi h^2\frac{dh}{dt}$

$\frac{dV}{dt} = \frac{4}{25}\pi h^2\frac{dh}{dt}$ ← of water

$2 = \frac{4}{25}\pi(5)^2\frac{dh}{dt}$

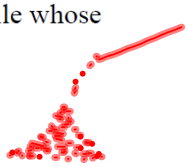
$2 = \frac{4}{25}\pi(25)\frac{dh}{dt}$

$2 = 4\pi\frac{dh}{dt}$

1 ft/

2) Corn is poured through a chute at the rate of $10 ft^3/min$, and falls in a conical pile whose bottom radius is always half the height.

$h = 2r$



(a) How fast will the radius of the base change when the pile is 8 ft high?

1. Given Information:

$\frac{dV}{dt} = 10\ ft^3/min$

$h = 8 \quad r = 4$

2. Looking for:

$\frac{dr}{dt}$

3. Equation:

$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi r^2 (2r)$

$V = \frac{1}{3}\pi 2r^3$

$V = \frac{2}{3}\pi r^3$

4. Use similar triangles to eliminate one variable, since we are looking for $\frac{dr}{dt}$, we solve for ___h___, so when we plug it into the volume equation the ___h___ goes away :

5. Take the derivative of the new equation

$V = \frac{2}{3}\pi r^3$

$\frac{dV}{dt} = 2\pi r^2 \frac{dr}{dt}$

$10 = 2\pi (4)^2 \frac{dr}{dt}$

$10 = 32\pi \frac{dr}{dt}$

$\frac{10}{32\pi} = \frac{dr}{dt}$

$\frac{10}{32\pi}\ ft/min$

(b) How fast is the circumference of the base increasing at this time?

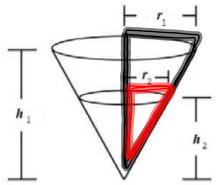
$C = 2\pi r$

$\frac{dC}{dt} = 2\pi \frac{dr}{dt}$ ← From Part (a)

$\frac{dC}{dt} = \cancel{2\pi}\left(\frac{10}{\cancel{32\pi}_{16}}\right)$

$\frac{dC}{dt} = \frac{10}{16} = \frac{5}{8}\ ft/min$

2) A conical water tank with a diameter of 20 ft at the top and is 24 feet high. If water flows into the tank at a rate of $20 ft^3/min$, how fast is the depth of the water increasing when the water is 16ft deep?



Know:

$d(\text{of tank}) = 20ft$

$r(\text{of tank}) = 10ft$

$h(\text{of tank}) = 24ft$

$\frac{dV}{dt} = 20 ft^3/min$

$h = 16ft$

Need:

$\frac{dh}{dt}$

$V = \frac{1}{3}\pi r^2 h$

$\frac{10}{24} = \frac{r}{h}$

$\frac{10}{24}h = r$

New Equation:

$$V = \frac{1}{3}\pi\left(\frac{10}{24}h\right)^2 h = \frac{1}{3}\pi\frac{100}{576}h^3 h = \frac{100}{1728}\pi h^3$$

$$V = \frac{100}{1728}\pi h^3$$

$$\frac{dV}{dt} = \frac{300}{1728}\pi h^2 \frac{dh}{dt}$$

$$20 = \frac{300}{1728}\pi (16)^2 \frac{dh}{dt}$$

$$20 = \frac{300}{1728}\pi (256)\frac{dh}{dt}$$

$$\frac{20 = \frac{76800}{1728}\pi\frac{dh}{dt}}{\frac{76800}{1728}}$$

$$\frac{9}{20} = \pi\frac{dh}{dt}$$

$$\frac{9}{20\pi} = \frac{dh}{dt}$$

$$\frac{9}{20\pi}\ ft/min = \frac{dh}{dt}$$

HW: Page 14 Cone question

$V = \frac{1}{3}\pi r^2 h$

KNOW:

$\frac{dV}{dt} = 10 \frac{ft^3}{min}$

$h(\text{gravel}) = 7ft$

$d = h$

$2r = h$

$r = \frac{h}{2}$

Need:

$\frac{dh}{dt}$

$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi \left(\frac{h}{2}\right)^2 h$

$V = \frac{1}{3}\pi \frac{h^2}{4} \cdot h$

$V = \frac{1}{3}\pi \frac{h^3}{4}$

$V = \frac{1}{12}\pi h^3$

$\frac{dV}{dt} = \frac{3}{12}\pi h^2 \frac{dh}{dt}$

$10 = \frac{1}{4}\pi (7)^2 \frac{dh}{dt}$

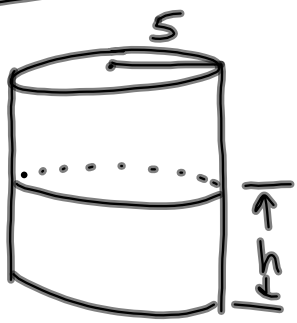
$\frac{4}{49} 10 = \frac{4}{49} \cdot \frac{49}{4} \pi \frac{dh}{dt}$

$\frac{40}{49} = \pi \frac{dh}{dt}$

$\frac{40}{49\pi} = \frac{dh}{dt}$

$\frac{40}{49\pi} \frac{ft}{min} = .260 \frac{ft}{min}$

AP Question:



Know:

$r = 5$

$\frac{dV}{dt} = -5\pi\sqrt{h}$

Need:

$\frac{dh}{dt} = -\frac{\sqrt{h}}{5}$

$V = \pi r^2 h$

$\frac{dV}{dt} = 2\pi r \frac{dr}{dt} h + \pi r^2 \frac{dh}{dt}$

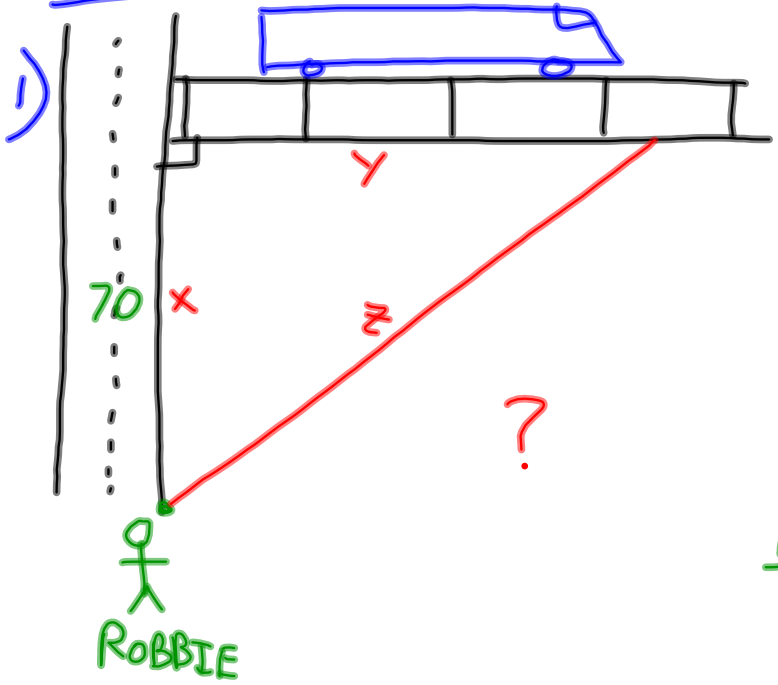
$-5\pi\sqrt{h} = \cancel{2\pi(5)(0)h} + \pi 5^2 \frac{dh}{dt}$

$\frac{\cancel{-5\pi}\sqrt{h}}{\cancel{25\pi}\ 5} = \frac{25\pi \frac{dh}{dt}}{25\pi}$

$-\frac{\sqrt{h}}{5} = \frac{dh}{dt}$

Review: → 60 m/s

1)



Know:

$x = 70$ $\frac{dx}{dt} = 0$

$y = 240$ $\frac{dy}{dt} = 60$

$z =$ $\frac{dz}{dt} =$

Need $\frac{dz}{dt}$